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THIN-FILM FLOW OF A QUASIVISCOUS FLUID OVER THE SURFACE OF A ROTATING NOZZLE

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Problems of the flow of a quasiviscous fluid with an arbitrary rheological law over the surface of a rapidly rotating nozzle are investigated.

The problem being investigated is of great practical value, since a thin-film flow of fluids over surfaces of rotating nozzles is often encountered in processes of dispersion, evaporation of solutions, mixing, absorption, scrubbing of gases, molecular distillation, etc. The flow of ordinary viscous fluids and fluids with quite complex rheological characteristics occurs in this case. One of the main parameters characterizing the flow process is the thickness of the fluid film on the surface of the rotating nozzle. Relationships for determining the thickness of the film of an ordinary viscous fluid [1-3] and for a non-Newtonian fluid only with a rheological power law [4] are known in the literature.

In this connection it is of interest to examine the problem of determining the thickness of the film of a quasiviscous fluid whose viscosity properties are expressed by an arbitrary rheological law in the form

$$\eta_e = f_1(\tau) = f_2(\dot{\gamma}). \tag{1}$$

The fluid is fed to the center of a rotating nozzle with a sharp edge and flows over it in a radial direction as a thin continuous laminar film under the effect of the centrifugal force. The movement of the fluid is examined in a cylindrical coordinate system r, φ, z rigidly associated with the center of the disk (Fig. 1). The following assumptions are made:

1) the flow of the fluid over the disk is steady;

2) the thickness of the fluid film is much less than the radius at which the flow is investigated, i.e., $\delta/r \ll 1$;

3) the effect of the gravitational force, surface tension forces, and frictional forces on the ambient medium is insignificant as a consequence of their smallness in comparison with the centrifugal forces.

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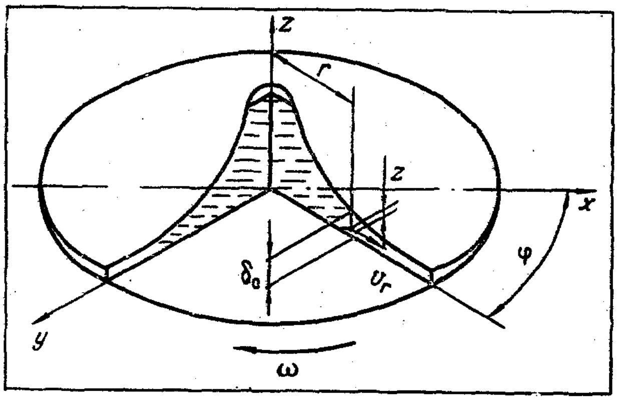

Fig. 1. Diagram of fluid flow.

The initial system of equations, obtained from [5], can be represented in the form

$$\omega_r = F_r - \frac{1}{\rho}\cdot\frac{\partial p}{\partial r} + \frac{1}{\rho}\left(\frac{\partial\tau_{rr}}{\partial r} + \frac{\partial\tau_{rz}}{\partial z} + \frac{\tau_{rr}}{r}\right),$$

$$\omega_z = F_z - \frac{1}{\rho}\cdot\frac{\partial p}{\partial z} + \frac{1}{\rho}\left(\frac{\partial\tau_{rz}}{\partial r} + \frac{\partial\tau_{zz}}{\partial z} + \frac{\tau_{rz}}{r}\right),$$

where

$$F = \omega^2 r,\ \tau_{rr} = 2\eta_e\varepsilon_{rr},\ \tau_{zr} = 2\eta_e\varepsilon_{zr},\ \tau_{zz} = 2\eta_e\varepsilon_{zz}. \tag{2}$$

With consideration of the assumptions made the equations of motion are written as

$$v_r\frac{\partial v_r}{\partial r} + v_z\frac{\partial v_r}{\partial z} = \omega^2 r - \frac{1}{\rho}\cdot\frac{\partial p}{\partial r} + \frac{1}{\rho}\cdot\frac{\partial}{\partial z}\left(\eta_e\frac{\partial v_r}{\partial z}\right), \tag{3}$$

$$\frac{\partial p}{\partial z} = 0. \tag{4}$$

The equation of continuity is

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{\partial v_z}{\partial z} = 0. \tag{5}$$

The boundary conditions for the given problem will have the form:

1. $v_r = v_z = 0$ when $z = 0$.
2. $\partial v_r/\partial z = 0$ when $z = \delta$.

It follows from (4) that $p = p_0 = \text{const}$ and, consequently, in Eq. (3), $\partial p/\partial r = 0$.

Taking into account that the inertial forces are considerably less than the viscous forces acting in the centrifugal field, we obtain

$$\frac{1}{\rho}\cdot\frac{\partial}{\partial z}\left(\eta_e\frac{\partial v_r}{\partial z}\right) = -\omega^2 r. \tag{6}$$

Multiplying the right and left sides of Eq. (6) by $v_r dz$, integrating from 0 to δ, and using the aforementioned boundary conditions, we represent Eq. (6) in the form

$$\int_0^\delta \eta_e\left(\frac{\partial v_r}{\partial z}\right)^2 dz = Q\,\frac{\rho\omega^2}{2\pi}, \tag{7}$$

where $Q = 2\pi r\int_0^\delta v_r dz$ is the volume flow rate of the fluid in unit time.

With consideration that $\tau = \eta_e(\partial v_r/\partial z)$, $d\tau = \rho\omega^2 r dz$ follows from (6). Then after a number of transformations we can represent Eq. (7) in the final form

$$\int_0^{\tau} \varphi\tau^2 d\tau = Q\,\frac{\rho^2\omega^4 r}{2\pi}\,. \tag{8}$$

Equation (8) establishes the relation between the flow rate and film thickness of the quasiviscous fluid with an arbitrary rheological law moving over the surface of a rapidly rotating nozzle.

As an example we can obtain from Eq. (8) the equation for determining the film thickness δ during flow of a quasiviscous fluid with a rheological power law of the type $\tau = k\dot{\gamma}^{n-1}\dot{\gamma}$.

Using dependence $d\tau = d\eta_e\dot{\gamma} + d\dot{\gamma}\eta_e$, we can represent Eq. (8) after a number of transformations as

$$\frac{n}{2n+1}\,k^2\left(\frac{\tau}{k}\right)^{\frac{2n+1}{n}} = Q\,\frac{\rho^2\omega^4 r}{2\pi}\,. \tag{9}$$

Taking into account that $\tau = \rho\omega^2 r\delta$, it is easy to obtain from relationship (9) the expression for thickness δ coinciding with the dependence obtained in [4].

For viscous fluids, assuming $\tau = \mu\dot{\gamma}$, from Eq. (8) we can obtain the known Hinze–Milborn dependence [1] for determining the film thickness.

As is known, there are more complex rheological equations of state describing the curve of the flow of the investigated fluids [6-8]. Investigations carried out by the authors on highly concentrated gelatin solutions showed the possibility of describing the complete flow curve by the following rheological equation:

$$\eta_e = \eta_\infty + \frac{\eta_0 - \eta_\infty}{(1+\alpha\dot{\gamma})^m} \tag{10}$$

and by the equation proposed by Cross [7]:

$$\eta_e = \eta_\infty + \frac{\eta_0 - \eta_\infty}{1+\beta\dot{\gamma}^n}\,. \tag{11}$$

In solving Eq. (8) with the use of rheological law (10) the following analytic expression is obtained by a number of transformations:

$$\begin{aligned} Q\,\frac{\rho^2\omega^4 r}{2\pi} = {} & \frac{\eta_\infty\dot{\gamma}^3}{3} + A_1\left\{\frac{m-2}{3-m}\left[(1+\alpha\dot{\gamma})^{3-m}-1\right] + \frac{4-3m}{2-m}\,\times\right. \\ & \times\left[(1+\alpha\dot{\gamma})^{2-m}-1\right] + \frac{3m-2}{1-m}\left[(1+\alpha\dot{\gamma})^{1-m}-1\right] + \left.\left[(1+\alpha\dot{\gamma})^{-m}-1\right]\right\} + \\ & + A_2\left\{\frac{1-m}{3-m}\left[(1+\alpha\dot{\gamma})^{3-2m}-1\right] + \frac{3m-2}{2-2m}\left[(1+\alpha\dot{\gamma})^{2-2m}-1\right] + \right. \\ & \left. + \frac{1-3m}{1-2m}\left[(1+\alpha\dot{\gamma})^{1-2m}-1\right] - \left[\frac{(1+\alpha\dot{\gamma})^{-2m}-1}{2}\right]\right\}, \\ & A_1 = \frac{\eta_\infty(\eta_\infty-\eta_0)}{\alpha^3},\quad A_2 = \frac{(\eta_\infty-\eta_0)^2}{\alpha^3}\,. \end{aligned} \tag{12}$$

Expression (12) permits calculating the theoretical value of the fluid film thickness as a function of the technological parameters of the process ω, r, ρ, Q and rheological properties of the fluid η_0, η_∞, α, m. Since the expression obtained is complex, Eq. (8) can be solved with the necessary accuracy by means of numerical methods on a computer according to the developed program. For this purpose the initial equation (8) with the use of dependence (10) can be represented as

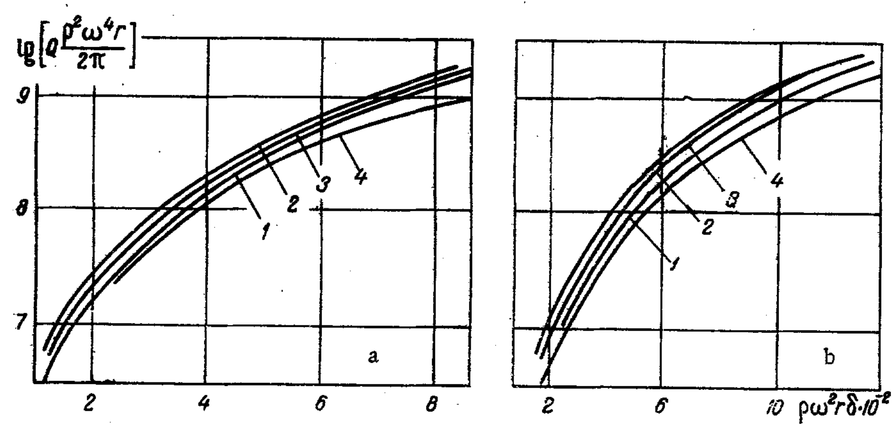

Fig. 2. Dependence of $Q(\rho^2\omega^4 r/2\pi)(N^2/sec\cdot m^4)$ on $\rho\omega^2 r\delta$ (N/m^2) for 18% (a) and 22% (b) gelatin solutions: 1) with the use of the fluidity values taken directly from the rheological curve; 2) according to Eq. (14); 3) according to Eq. (13); 4) according to the Hinze–Milborn formula.

$$Q\,\frac{\rho^2\omega^4 r}{2\pi} = \int_0^{\dot\gamma_{max}} \left\{\left[\eta_\infty + \frac{\eta_0-\eta_\infty}{(1+\alpha\dot\gamma)^m}\right]\dot\gamma^3\, m\alpha\,(\eta_\infty-\eta_0)\times\right.$$
$$\left.\times(1+\alpha\dot\gamma)^{-m-1}\right\}d\dot\gamma + \int_0^{\dot\gamma_{max}} \left[\eta_\infty + \frac{\eta_0-\eta_\infty}{(1+\alpha\dot\gamma)^m}\right]^2 \dot\gamma^2\,d\dot\gamma. \tag{13}$$

With the use of the Cross rheological equation we can represent Eq. (8) in the following form convenient for solving by the numerical method on a computer:

$$Q\,\frac{\rho^2\omega^4 r}{2\pi} = \int_0^{\dot\gamma_{max}} \left(\eta_\infty + \frac{\eta_0-\eta_\infty}{1+\beta\dot\gamma^n}\right)\frac{n\beta\,(\eta_\infty-\eta_0)}{(1+\beta\dot\gamma^n)^2}\,\dot\gamma^{n+2}\,d\dot\gamma + \int_0^{\dot\gamma_{max}} \left(\eta_\infty + \frac{\eta_0-\eta_\infty}{1+\beta\dot\gamma^n}\right)^2\dot\gamma^2\,d\dot\gamma. \tag{14}$$

Along with the solutions of (13), (14) we obtained the solution of Eq. (8) with the use of the fluidity values taken directly from the experimental curves.

The theoretical data obtained on the basis of solving Eqs. (13), (14), and Eq. (8) with the use of the experimental fluidity values are presented as graphs of the dependences in coordinates $Q\rho^2\omega^4 r/2\pi$–$\rho\omega^2 r\delta$ (Fig. 2a, b). The calculated values of the film thickness were determined from the relationship

$$\delta = \frac{\tau}{\rho\omega^2 r}. \tag{15}$$

The calculated values of the film thickness were compared with the experiments to check the reliability of the proposed equations.

The experiments were carried out on a flat rotating disk with a diameter of 140 mm having a polished surface and sharp spraying edge with the use of 10, 15, 18, and 22% gelatin solutions at a temperature of 45°C. The flow rate of the fluid was varied from $6.3\cdot 10^{-6}$ to $31\cdot 10^{-6}$ m^3/sec and the number of revolutions from 1000 to 2860 rpm.

To measure the film thickness we used a device that is based on the use of the electrical conductivity of the investigated fluids and is known from [4, 9].

The average divergence of the theoretical and experimental values of the film thickness was 5-8% according to Eq. (13), 7-10% according to Eq. (14), and 7% with the use of the fluidity values taken directly from the experimental curve.

For comparison the graph (Fig. 2) shows the values of the film thickness calculated by the Hinze–Milborn dependence for viscous fluids [1]. The maximum divergence of the theoretical and experimental data ranges from 10 to 25%.

NOTATION

η_e, effective viscosity, N•sec/m²; τ, internal friction stress, N/m²; $\dot{\gamma}$, shear velocity gradient, 1/sec; k, n, m, α, β, rheological constants; η_0, first Newtonian viscosity, N•sec/m²; η_∞, viscosity of the ultimately destroyed structure, N•sec/m²; ε_{rr}, ε_{rz}, ε_{zz}, strain-rate components; ρ, density, N•sec²/m⁴; F_r, F_z, projections of mass forces in direction r and z; ω, angular velocity of disk rotation, 1/sec; p, pressure, N/m²; δ, thickness of fluid film; r, instantaneous radius of disk, m; v_r, v_z, velocity components of fluid in respective directions r and z, m/sec; φ, fluidity of fluid; Q, volume flow rate of fluid, m³/sec.

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ELECTROCHEMICAL METHOD OF STUDYING HEAT TRANSFER BETWEEN A CYLINDER AND A RISING TWO-PHASE FLOW

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An electrochemical method is used to study the mean coefficients of mass transfer, and it is shown that the experimental data are in complete agreement with a relationship [Eq. (2)] derived from experiments with thermal models.

The results of an experimental investigation into heat transfer between a transversely flushed cylinder and an equilibrium ($\varphi = 1$) rising gas flow containing drops of liquid were presented in [1]. In this problem the volumetric concentration of the suspended drops, i.e., the ratio of the volume of the drops to the volume occupied by the gas, was no greater than $5 \cdot 10^{-2}$. Depending on the concentration of the drops in the two-phase flow, and hence the amount of liquid deposited on the surface of the cylinder, heat transfer occurred both as a result of the heating of the film and its detachment from the surface (q_{de}) and as a result of the heat and mass transfer between the surface and the flow (q_{con} and q_{ev}).

For low concentrations of liquid in the flow, the surface temperature of the liquid film exceeds the equilibrium temperature of the flow, and heat is largely carried away by processes of heat transfer (nonevaporating liquid) or heat and mass transfer (evaporating liquid) between the film and the flow.

For high concentrations the surface temperature of the film is approximately equal to the equilibrium temperature of the flow, and the heat from the cylinder is carried away by convection between the flowing film and the surface of the cylinder, i.e., by the heating and subsequent detachment (separation) of the film.

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